BINOMIAL THEOREM

ABDUL AHAD M

SUBJECT MATTER EXPERT - MATHS

SECTION - 1. INTRODUCTION TO BINOMIAL THEOREM

TOPIC - 1. BINOMIAL & PASCAL'S TRIANGLE

Pre-requisites

- Knowledge about the polynomials and the different types based on the number of terms.
- Basic understanding of algebraic expressions, terms and coefficients.

Introduction

What are polynomials? Different types of polynomials based on the number of terms, how a pascal's triangle is formed, how the coefficients of terms in a binomial expansion can be found out using the Pascal's triangle. These questions will be answered in the following.

Classification of polynomials

- Based on the number of terms

  - **Monomial**

    A type of polynomial which is an algebraic expression having only one term. It can have multiple variables and higher powers also.

    Example : $5x^4y^7z$ is a monomial with different variables, x, y and z.

    Same goes for the monomials like x^3, y^5, xz^{20} etc.

  - **Binomial**

    The algebraic expressions which contain two terms are called binomials. It can also be defined as the sum or difference of two separate monomials

    Example : $ax^p \pm by^q$ is a binomial as it contains only 2 terms, or we can even say its a combination of two separate monomials, ax^p and by^q.

    Other examples include : $2x^6 - 7y^2, 5abc + 3a^3c$ etc.

  - **Trinomial**

    The algebraic expressions which contain three terms are called trinomials.

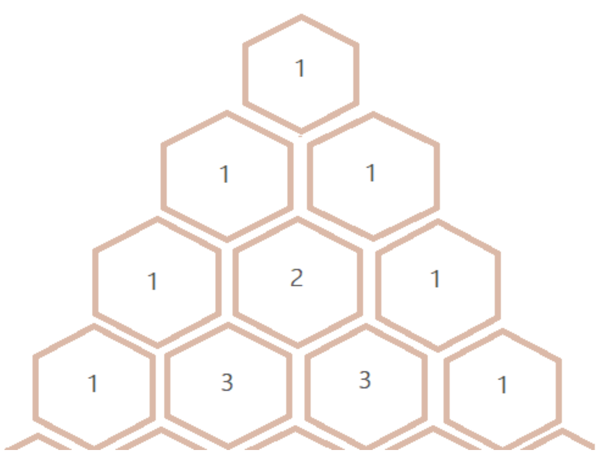
It can also be defined as the sum of difference between three separate monomials.

Example : $3x^2 + 5y^2 + z$, $t^5 + 6t^3 + t$ . . . All these contains three terms connected by $+, -$.

Pascal's triangle

BLAISE PASCAL 1623 ,1662

In algebra, a triangular arrangement of numbers that gives the coefficients in the expansion of any binomial expression. Then the triangle can be filled out from the top by adding together the two numbers just above to the left and right of each position in the triangle.



As you can see the pascal's triangle, the end/ sides of the triangle are all having values 1. Then subsequently as we come down, look at the 3rd row; the middle value is the sum of the values in the adjacent boxes in the top row. Same as we go down each row. This way we can find out the coefficients of each binomial expansion with powers starting from 0.

$(x + y)^0 = 1$	the coefficient is	**1**			
$(x + y)^1 = x + y$	the coefficients are	**1**	**1**		
$(x + y)^2 = x^2 + 2xy + y^2$	the coefficients are	**1**	**2**	**1**	
$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$	the coefficients are	**1**	**3**	**3**	**1**

and so on . . . .

Also note that the powers of x **decreases** as we move right and the powers of y **increases** when we move right. Here we can see the power of x starts with **3** and then decreases to **0**. Same way the power of y starts with **0** and then increases to **3**. This pattern is followed in all the binomial expansions.

Using this pascals triangle, we can find out the coefficients of the binomial expansions with positive integral powers. For eg: $(x + y)^4$

The coefficients of this expansion will be the 5th level of the triangle, ie

1 4 6 4 1

Therefore the expansion $(x + y)^4$ can be written as $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

Coefficients of binomial expansion can also be easily determined by a triangular array of binomial coefficients constructed by summing binomial coefficients of adjacent elements in preceding rows.

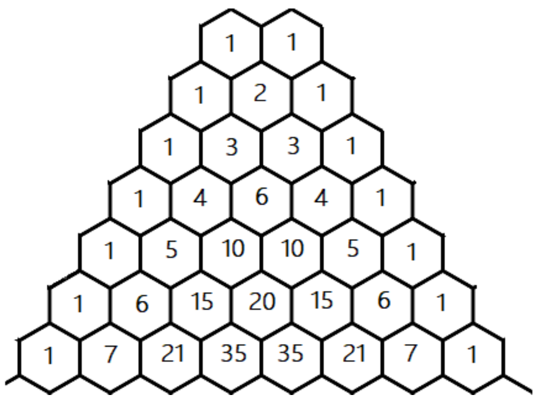
Examples

I. Find the expansion of $(a + b)^7$ using Pascal's triangle.

Analysis : Here they have asked us to find out the expansion of a binomial $(a + b)$ to the power 7. To do that let’s first look at the Pascal's triangle and the way the triangle is being formed.

Solution : $(a + b)^7$ is asked here.

Let's construct the Pascal's triangle and see what the coefficients will be.



From the above pascals triangle, it is clear that the last row with numbers

1 7 21 35 35 21 7 1 should be the coefficients of the expansion $(a + b)^7$

And also remember about the change in the powers. The power of a decreases and the power of b increases.

Therefore the expansion can be written as

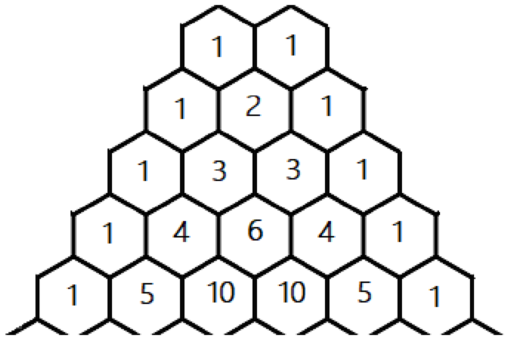
$$(a + b)^7 = a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$$

II. Find the expansion of $(x - 2y)^5$ using Pascal's triangle.

Analysis : Based on how we did the previous question, now you might have got an idea about how to find the expansion given in the question here. But there are some few changes that you can observe here.

- The last question was addition, here you can see the terms are connected by subtraction.
- The coefficient of both terms in the previous expansion was 1, here the coefficient of y is 2 or taking the minus sign together, we can say its -2

Solution : In the expansion $(x - 2y)^5$ we can see the power is 5. So let's use the Pascal's triangle to first find out the coefficients of the terms in this expansion.



The powers of this expansion will contain the coefficients

1 5 10 10 5 1

Look at the second term here, ie '-2y' and we also know that the power of the first term decreases as we move right and the second term power increases.

So $(x - 2y)^5$ can be written as

$x^5 + 5x^4(-2y) + 10x^3(-2y)^2 + 10x^2(-2y)^3 + 5x(-2y)^4 + (-2y)^5$

$= x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

Summary

- Types of polynomials based on the number of terms were revisited.
- Focus about the binomials.
- Pascal's triangle introduced
- How to find the coefficients of terms in an expansion using Pascal's triangle.
- How the power of the terms varies in an expansion.
- What happens when there is a negative term in the expansion.
- 2 examples based on the topics we studied.